

Dynamical Localization of Gravity

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Abstract

We show that the thin wall limit of the thick domain wall associated with a sine-Gordon soliton in a single non-compactified patch of 5-dimensional spacetime explicitly yields the Randall-Sundrum localized gravity two patch brane, with its discrete Z_2 symmetry arising from the discrete symmetry of the potential, and with the thin Minkowski brane $\Lambda_5 + \kappa_5^2 \Lambda_b^2/6 = 0$ relation between bulk and brane cosmological constants arising naturally without any need for fine tuning. Additionally we show that for an embedded thin de Sitter brane, localization of gravity is again possible provided the 5-space is compactified, with the now non-zero net cosmological constant $\Lambda_5 + \kappa_5^2 \Lambda_b^2/6$ on the brane being found to vary inversely with the compactification radius.

Recently Randall and Sundrum [1,2] have shown that it is possible for our 4-dimensional universe to be a brane embedded in a 5-dimensional bulk spacetime whose spacelike extra dimension need not in fact be tiny. Intrinsic to their study was a Z_2 symmetry associated with the piecing together of two separate patches of AdS_5 spacetime at the brane in a way which then led to exponential suppression of gravity on both sides of the brane. It is thus of interest to find an origin for this otherwise simply presupposed Z_2 symmetry, and in this paper we will show that both it and the emergence of the two separate AdS_5 patches can arise naturally from the thin wall limit of the topological domain wall which is generated by a gravitationally supported soliton mode associated with a sine-Gordon scalar field ϕ in a curved 5-dimensional spacetime background.

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In order to eventually make contact with the R_4 Minkowski thin brane case studied by Randall and Sundrum (RS) as well as try to generalize their result to de Sitter and anti de Sitter branes, we shall consider the embedding of an arbitrary maximally symmetric 4-space (viz. one not yet a brane) into an arbitrary (viz. one not yet AdS_5) 5-space, with the most general metrics for such embeddings of the three possible 4-spaces being given by

$$\begin{aligned} ds^2(R_4) &= dw^2 + e^{2f}[dx^2 + dy^2 + dz^2 - dt^2], \\ ds^2(dS_4) &= dw^2 + e^{2f}[e^{2at}(dx^2 + dy^2 + dz^2) - dt^2], \\ ds^2(AdS_4) &= dw^2 + e^{2f}[dx^2 + e^{2bx}(dy^2 + dz^2 - dt^2)], \end{aligned} \quad (1)$$

where in each case $f(w)$ is an arbitrary function of the extra coordinate w , one which need not even be a symmetric function of w , let alone be one of the $f = -|w|$ form required in the RS R_4 Minkowski case. For the 5-space we shall take the gravitational equations of motion to be of the form

$$G_{AB} = R_{AB} - g_{AB}R^C_C/2 = -\kappa_5^2 T_{AB} \quad (2)$$

where T_{AB} ($A, B = 0, 1, 2, 3, 5$) is due only to a bulk scalar field ϕ with yet to be specified $\phi \rightarrow -\phi$ invariant potential $V(\phi)$, so that the field equations then take the form:

$$\begin{aligned} 3f'' + 6f'^2 - 3\sigma e^{-2f} &= -\kappa_5^2 e^{-2f} T_{00} = -\kappa_5^2 [\phi'^2/2 + V(\phi)], \\ 6f'^2 - 6\sigma e^{-2f} &= \kappa_5^2 T_{55} = \kappa_5^2 [\phi'^2/2 - V(\phi)], \\ \phi'' + 4f'\phi' &= dV(\phi)/d\phi, \end{aligned} \quad (3)$$

where $\sigma = 0, a^2, -b^2$ in the three cases under discussion. As regards these equations we note immediately that independent of the explicit structure of the potential, as long as the theory supports a kink or soliton which interpolates between different degenerate vacua of $V(\phi)$, $\phi(w)$ will then be an odd function of w , thus causing $f(w)$ to be an even function of w . A Z_2 type symmetry of a potential thus translates directly into a Z_2 symmetry of the geometry. Spontaneous symmetry breaking thus naturally yields the RS Z_2 .

Recalling the geometric connection between maximally symmetric spaces and the sine-Gordon equation,¹ and recalling that our sought AdS_5 is itself a space of constant negative curvature, it is thus suggested to try as an explicit potential

$$V(\phi) = A^2\beta^2/8 - (A^2\beta^2/8)(1 + \kappa_2^5 A^2/3)\sin^2(2\phi/A). \quad (4)$$

On setting

$$\tan(\phi/A) = \tanh(\beta w/2), \quad \phi' = (A\beta/2)\operatorname{sech}(\beta w) = (A\beta/2)\cos(2\phi/A), \quad (5)$$

¹The general (Euclidean and Minkowski) signed 2-dimensional metrics $ds^2 = dx^2[1 + \cos\theta(x, y)] \pm dy^2[1 - \cos\theta(x, y)]$ will be spaces of constant 2-curvature K provided (see e.g. [3]) θ obeys the conditions $\partial^2\theta/\partial_x^2 \mp \partial^2\theta/\partial_y^2 = -2K\sin\theta$, and will support the y -independent soliton $\tan(\theta/4) = \exp[(-2K)^{1/2}x]$ if K is negative.

viz. a solitonic thick domain wall mode, we find that an exact solution to the entire set of field equations of Eq. (3) then obtains² in the $\sigma = 0$ R_4 case provided (D is another constant)

$$f' = -(A^2\beta\kappa_5^2/12)\tanh(\beta w), \quad e^f = D[\cosh(\beta w)]^{-A^2\kappa_5^2/12}, \quad (6)$$

a solution we recognize as precisely being of a thick brane form. While not being an AdS_5 metric,³ e^f is nonetheless seen to peak at $w = 0$ and to fall exponentially to zero at $w = \pm\infty$, to thus rapidly localize the geometry to the $w = 0$ region, without any need for the domain wall to be thin. Since fluctuations of the form $ds^2 = dw^2 + e^{2f}dx^\mu dx^\nu(\eta_{\mu\nu} + \psi_{\mu\nu})$ around this metric obey [1,2,6,7] the covariant 5-space scalar wave equation associated with Eq. (1), viz.

$$(\partial_w^2 + 4f'\partial_w)\psi_{\mu\nu} = -e^{-2f}(\partial_x^2 + \partial_y^2 + \partial_z^2 - \partial_t^2)\psi_{\mu\nu} = -e^{-2f}m^2\psi_{\mu\nu}, \quad (7)$$

where m^2 is a separation constant, we see that no matter what the explicit form of $f(w)$, the theory will admit of an $m^2 = 0$ massless 4-dimensional graviton mode with an associated metric fluctuation $h_{\mu\nu}(w) = e^{2f}\psi_{\mu\nu}$ which behaves as e^{2f} . Localization of the geometry to the $w = 0$ region thus entails localization of gravity to the $w = 0$ region as well, with $f(w)$ thus not needing to be restricted to the particular $e^{-|w|}$ AdS_5 RS fall-off in order to localize, with Eq. (6) providing a far more general such option.⁴

As far as gravity alone is concerned, a macroscopically sized (though not too thick) brane could actually be phenomenologically acceptable. However, in such a non delta function thin case the $SU(3) \times SU(2) \times SU(1)$ particle physics fields could then potentially leak into the fifth dimension as well, with it being particle physics which thus obliges us to look for a thin brane limit of our above model. Moreover, it is important to stress that the localization associated with the metric of Eq. (6) has been obtained above with the use of only one single 5-space coordinate patch which embraces both positive and negative w , whereas the particle physics required RS thin brane entails the presence of two. It is thus very interesting to now note that our thick brane solution can precisely be brought to the RS thin brane two patch form by taking a very delicate limit in which we let A go to zero and β go to infinity while holding $A^2\beta$ fixed. Noting that in the $\beta \rightarrow \infty$ limit the quantity $[\cosh(\beta w)]^{-1/\beta} \rightarrow e^{-|w|}$, we thus see that $f \rightarrow -\kappa_5^2 A^2 \beta |w|/12$, i.e. precisely to the standard RS form. With the solitonic ϕ becoming a step function in this same limit, we see that we generate a thin domain wall

²This solution was also reported in [4,5] where some interesting implications for brane localized gravity were presented.

³With a spatially varying scalar field yielding an energy-momentum tensor which is not purely a 5-dimensional cosmological constant, the bulk geometry could not be AdS_5 .

⁴In fact, any metric which can be written in the generic separable $ds^2 = dw^2 + e^{2f(w)}dx^\mu dx^\nu q_{\mu\nu}(x, y, z, t)$ form will admit of a separable analog of Eq. (7) and thus necessarily support an $m^2 = 0$ mode which will move on the light cone associated with the 4-space metric $q_{\mu\nu}(x, y, z, t)$. It is only the relative importance of the $m^2 > 0$ modes, or the possible existence of tachyons, which would be sensitive to the explicit form of $q_{\mu\nu}$.

in the limit, with this wall becoming a brane which then divides our original one coordinate patch into two now disconnected regions across which there is a discontinuous jump in the extrinsic curvature $K_{\mu\nu} = \eta_{\mu\nu} f'$ of the brane. The solitonic mode thus generates the RS thin brane dynamically,⁵ to not only recover it, but to also put it on a far more secure theoretical foundation, one based on structures quite familiar in particle physics (the higher dimensional ϕ itself might even be related to a string theory dilaton).

On identifying the value of the sine-Gordon potential at its degenerate minima as $V(\pm\pi A/4) = \Lambda_5 = -\kappa_5^2 A^4 \beta^2 / 24$, we thus see that $f \rightarrow -(-\Lambda_5 \kappa_5^2 / 6)^{1/2} |w|$ in the brane limit, so that $f' \rightarrow -(-\Lambda_5 \kappa_5^2 / 6)^{1/2} \epsilon(w)$ (here $\epsilon(w) = \theta(w) - \theta(-w)$), while $f'' \rightarrow -2(-\Lambda_5 \kappa_5^2 / 6)^{1/2} \delta(w)$, to thus generate jump and delta function singularities at the brane. In order to determine exactly where this delta function comes from in our model, we note from Eq. (3) that the simultaneous presence of a delta function singularity in f'' and the absence of one in f' entail that $\phi'^2/2$ and $V(\phi)$ must both contain the same delta function. And indeed from Eq. (5) we see immediately that $\phi'^2 \rightarrow A^2 \beta \delta(w)/2$ in the limit, to thus generate a term $e^{2f} [\phi'^2/2 + V(\phi)] \rightarrow (A^2 \beta / 2) \delta(w)$ in T_{00} , an effective brane cosmological constant term $\Lambda_b \delta(w) = (A^2 \beta / 2) \delta(w)$ which then automatically obeys the condition $\Lambda_5 + \kappa_5^2 \Lambda_b^2 / 6 = 0$. Thus what must be thought of as a fine tuning condition in an RS theory with an a priori brane possessing a Λ_b which is introduced by hand is now seen as being a dynamical output to a soliton induced brane, with Λ_b being due to the energy density of the bulk soliton when it is squeezed to the selfsame brane which it itself generates.

It is also instructive to derive this effective Λ_b in a slightly different manner. Specifically, we can also define it globally, with the action $-\int dw g^{1/2} L = \int dw e^{4f} [\phi'^2/2 + V(\phi)] = -3 \int dw e^{4f} (f'' + 2f'^2) / \kappa_5^2$ being required to recover the RS action $\int dw e^{2f} [e^{2f} \Lambda_5 + \Lambda_b \delta(w)]$ in the thin brane limit. Explicit calculation then shows that this is indeed the case while revealing that the Λ_b term comes entirely from the f'' term according to $\Lambda_b = A^2 \beta / 2$, with the Λ_5 term coming entirely from the f'^2 term according to $\Lambda_5 = -A^4 \beta^2 \kappa_5^2 / 24 = -\kappa_5^2 \Lambda_b^2 / 6$. Moreover, not only does Λ_b come purely from the f'' integral, in the limit the integral evaluates to $\Lambda_b = -3[f'(+\infty) - f'(-\infty)] / \kappa_5^2$. With f' being given by $-(\kappa_5^2 A^2 \beta / 12) \tanh(\beta w) \rightarrow -(\kappa_5^2 A^2 \beta / 12) \epsilon(w)$, we may thus interpret the induced $\Lambda_b = A^2 \beta / 2$ as a topological charge.⁶

Turning now to the dS_4 and AdS_4 embedded cases, we note first that while the general 5-dimensional metrics of Eqs. (1) are not necessarily AdS_5 metrics, requiring their associated

⁵For an earlier attempt to generate a brane using the $\phi = A \tanh(\beta w)$ kink mode see [7].

⁶Since Eq. (1) describes the general embedding of a 4-space into a 5-space, already before taking any thin brane limit we may set $K_{\mu\nu}(\infty) - K_{\mu\nu}(-\infty) = \eta_{\mu\nu} [f'(\infty) - f'(-\infty)]$. Since we then obtain $f'(\pm\infty) = f'(0^\pm)$ in the ensuing thin brane limit, we may thus write $K_{\mu\nu}(\infty) - K_{\mu\nu}(-\infty) = \eta_{\mu\nu} [f'(0^+) - f'(0^-)] = K_{\mu\nu}(0^+) - K_{\mu\nu}(0^-)$. Then, since the discontinuity at the brane obeys the Israel junction condition $K_{\mu\nu}(0^+) - K_{\mu\nu}(0^-) = -\kappa_5^2 (T_{\mu\nu} - q_{\mu\nu} T_\alpha^\alpha / 3) = -\kappa_5^2 \eta_{\mu\nu} \Lambda_b / 3$, we can thus, and with full generality, identify Λ_b as the topological $f'(\infty) - f'(-\infty) = -\kappa_5^2 \Lambda_b / 3$. As a measure of the universality of such a topological charge, we note in passing that the sextic potential $V(\phi) = A^2 \beta^2 (1 - \phi^2 / A^2)^2 / 2 - 2\kappa_5^2 A^2 \beta^2 \phi^2 (3 - \phi^2 / A^2)^2 / 27$ (a potential also considered in [7]) admits of the exact solution $\phi = A \tanh(\beta w)$, $e^f = \exp[A^2 \kappa_5^2 / 18 \cosh^2(\beta w)] \cosh(\beta w)^{-2A^2 \kappa_5^2 / 9}$, a solution which also possesses an RS brane limit in which $\Lambda_b = (-6V_{min} / \kappa_5^2)^{1/2} = -3[f'(+\infty) - f'(-\infty)] / \kappa_5^2$.

Einstein tensors to be given as their respective metric tensors⁷ would make them so. For R_4 this leads to $e^f = e^{\pm w}$, for dS_4 to $e^f = \sinh w$ and for AdS_4 to $e^f = \cosh w$. With both of the dS_4 and AdS_4 metrics thus growing at both large positive and large negative w , prospects for localization of gravity in these cases initially appear to be somewhat slim.⁸ In order to see whether these cases could anyway support a brane (regardless of any possible localization of gravity considerations), the natural thing to do is consider metrics of the forms $e^f = \sinh|w|$ and $e^f = \cosh|w|$. However, neither of these cases works as such since even while their Einstein tensors now contain $\delta(w)$ terms, the coefficients of these delta functions respectively diverge and vanish at $w = 0$. However, it is possible to avoid these difficulties by simply shifting the position of these singularities elsewhere, to thus instead take $e^f = \sinh(\gamma - |w|)/\sinh\gamma$ and $e^f = \cosh(\gamma - |w|)/\cosh\gamma$ where γ is a constant.⁹ While the choice of sign of the shift parameter γ is arbitrary, we note that for positive γ both of the metrics would then have a local maximum at $w = 0$. Thus, if the metrics could be cut-off at $w = \pm\gamma$ by compactifying the fifth dimension into a circle via an identification of $w = +\gamma$ with $w = -\gamma$, gravity would then fall off on both sides of the brane.¹⁰ With the dS_4 metric actually vanishing at the compactification radius, and with derivatives of $f(w)$ diverging there, the very existence of such singular behavior could cause the $|w| > \gamma$ region to be cut-off, and thus provide a mechanism for compactification in the first place, to thus make dS_4 a potentially promising candidate for a possible such self compactification.

While we will explicitly explore this possibility in detail below, before doing so however, we find it instructive to first consider the $e^f = \sinh(\gamma - |w|)/\sinh\gamma$ and $e^f = \cosh(\gamma - |w|)/\cosh\gamma$ cases in their own right simply as RS type thin brane theories with given a priori $T_{00} = e^{2f}\Lambda_5 + \Lambda_b\delta(w)$, $T_{55} = -\Lambda_5$ cosmological constant sources. In such a situation both of these metrics are then found to be exact RS type solutions to the Einstein equations, with their various parameters being related according to

$$a^2 \sinh^2 \gamma = -\kappa_5^2 \Lambda_5 / 6 = 1, \quad \Lambda_5 + \kappa_5^2 \Lambda_b^2 / 6 = 6a^2 / \kappa_5^2 > 0 \quad (8)$$

in the dS_4 case, and to

⁷With the Weyl tensor vanishing identically for arbitrary $f(w)$ for each of the three metrics in Eqs. (1), setting each Ricci tensor equal to the relevant metric tensor makes each such metric maximally 5-space symmetric.

⁸Exactly this same situation is encountered in the embedding of maximally 3-symmetric Robertson-Walker branes in a general 5-space [8] or even into an AdS_5 one [9].

⁹The thin brane $e^f = \sinh(\gamma - |w|)$ metric was also considered in [10,7,11], with discussion of a thick dS_4 brane being given in [12].

¹⁰Since the geometry would fall off exponentially on both sides of the brane for $|w| \ll \gamma$, and since both the \sinh and \cosh are monotonic functions of their arguments, the geometry would then fall off all the way to $w = \pm\gamma$, so that the compactification radius γ need not be as tiny as required in standard compactified empty bulk Kaluza-Klein theories in which the 5-dimensional geometry is taken to be the compactified but otherwise flat $S_1 \times R_4$.

$$b^2 \cosh^2 \gamma = -\kappa_5^2 \Lambda_5 / 6 = 1, \quad \Lambda_5 + \kappa_5^2 \Lambda_b^2 / 6 = -6b^2 / \kappa_5^2 < 0 \quad (9)$$

in the AdS_4 one. From the point of view of pure RS theory then, we see that when the brane Λ_b does not obey the $\Lambda_5 + \kappa_5^2 \Lambda_b^2 / 6 = 0$ R_4 fine tuning condition, the brane must instead be dS_4 or AdS_4 according to whether the excess or residual brane cosmological constant $\Lambda_b - (-6\Lambda_5 \kappa_5^2)^{1/2}$ is positive or negative.¹¹ Thus no matter what the value of Λ_b , there will always be an appropriate associated maximally 4-symmetric brane, so that the RS fine-tuning condition does not in fact need to be imposed even in theories with a priori branes. Rather, the brane topology adjusts itself according to whatever value Λ_b has. Moreover, since $6a^2 / \kappa_5^2 = 6 / \kappa_5^2 \sinh^2 \gamma$, we see in the event of any such excess, that the residual brane cosmological constant would then vary inversely with the compactification radius.¹² Consequently, a large compactification radius (something now possible with localized gravity) entails a small residual brane cosmological constant. While this is a very interesting such correlation, it is important to note that this does not yet constitute a solution to the cosmological constant problem until some independent reason is identified which would lead to a large compactification radius in the first place, since a large particle physics scale cosmological constant on the brane would itself otherwise lead only to a small compactification radius.¹³

In trying to construct the above dS_4 brane as a limiting solution to a 5-dimensional scalar field theory, we shall try to parallel the Minkowski case brane discussion by looking for a soliton like solution in which the scalar field goes to separate degenerate minima, $\phi(\pm\gamma)$, at the two compactification points, with ϕ' vanishing at those points.¹⁴ However now, given the singularity structure of the $e^f = \sinh(\gamma - |w|) / \sinh \gamma$, $f' = -\epsilon(w) \coth(\gamma - |w|)$ metric at $w = \pm\gamma$, we need to determine how a singular behavior such as this (assuming it to be present prior to taking the brane limit) would affect the scalar field at those points. Thus, taking the scalar field to behave as $\phi(w) = \phi(\gamma) + E(\gamma - w)^n$ near $w = \gamma$, then entails that $V'(\phi)$ must behave as $\phi'' + 4f'\phi' = En(n+3)(\gamma - w)^{n-2}$ near the compactification points. If $V(\phi)$ is to be

¹¹With the fine-tuned R_4 RS value for Λ_b arising from a higher dimensional soliton, this excess brane cosmological constant could be thought of as being due to the contribution of the ordinary 4-dimensional particle physics fields on the brane; and with these latter fields also obeying curved space wave equations such as the equation $[\partial_w^2 + 4f'\partial_w + e^{-2f}\nabla^2]\psi = 0$ discussed above, we thus note that these latter fields will also be confined to the brane by the 5-dimensional soliton.

¹²A similar result was also noted in [11] in a two brane model in which the branes were located at the $w = \pm\gamma$ compactification points.

¹³For a completely different approach to the cosmological constant problem, a strictly 4-dimensional one in which the cosmological constant can have acceptably small observable consequences for 4-dimensional physics even when being as big as particle physics suggests see [13].

¹⁴In passing we note that the field $\tan\phi = 2\gamma w / (\gamma^2 - w^2)$ obeys $\phi' = 2\gamma / (\gamma^2 + w^2) = (1 + \cos\phi) / \gamma$, and is thus actually associated with a compactified flat space sine-Gordon equation. However, in such a configuration ϕ' is non-vanishing at the compactification points, and thus not of interest for our purposes here.

a well-behaved, Taylor series expandable, potential with a smooth minimum at $w = \gamma$, then it must behave near such a minimum as $V''(\phi(\gamma))[\phi - \phi(\gamma)] = EV''(\phi(\gamma))[\gamma - w]^n$, a form which we see is simply not compatible with $\phi'' + 4f'\phi'$ no matter what the value of n . Hence, near its minima $V(\phi)$ cannot in fact be as well-behaved as a standard particle physics Higgs potential. Rather, $V'(\phi)$ must also behave as $(\gamma - w)^{n-2}$, i.e. as $[\phi(\gamma) - \phi]^{(n-2)/n}$, so that the (necessarily real) potential itself must behave as the non-analytic $(|\phi - \phi(\gamma)|)^{2(n-1)/n}$ and thus have a cusp at $w = \gamma$. With the same behavior having to occur at $w = -\gamma$ as well, rather than being a smooth double well potential, $V(\phi)$ would have to be a double or periodic ($\simeq \sin|\phi|$) cusp potential, with the loss of analyticity at the two cusp minima signaling the presence of the singular compactification points, while also potentially providing us with a mechanism to disconnect the $|w| < \gamma$ and $|w| > \gamma$ regions (possibly even by putting additional branes at the compactification points).

Unfortunately, for the moment, we have so far been unable to find any exact solutions to the theory which have this particular cusp structure at the compactification points and then produce a brane at $w = 0$ when the brane limit is taken. However, we have, instead, been able to find a different type of solution, one with another kind of singularity structure, viz. one in which the scalar field itself also diverges at the compactification points. And despite the fact that this is at first somewhat disquieting requirement, we shall now show that not only is it feasible, but that it is even achievable without any divergence in the energy density. In fact, we have actually found a closed form solution to the field equations at the brane limit, one which explicitly allows a divergent ϕ to support a dS_4 brane. Specifically, the field configuration (viz. a configuration for which $\phi(w = \pm\gamma) = \pm\infty$)

$$e^{2\phi/\nu} = \epsilon(w)[\cosh(\gamma - |w|) + 1]/[\cosh(\gamma - |w|) - 1] - e^{2\mu/\nu}, \quad e^f = \sinh(\gamma - |w|)/\sinh\gamma \quad (10)$$

(here $e^{2\mu/\nu} = \epsilon(w)[\cosh\gamma + 1]/[\cosh\gamma - 1]$) is found to be an exact solution to the field equations when the potential is taken to have the "shine-Gordon" form¹⁵

$$V = V_0 - 3\nu^2 \sinh^2(\phi/\nu + \mu/\nu)/2, \quad (11)$$

and T_{00} is taken to also include an explicit excess brane cosmological constant according to $e^{-2f}T_{00} = \phi'^2/2 + V(\phi) + \Lambda_b\delta(w)$ ($T_{55} = \phi'^2/2 - V(\phi)$ is taken to be unchanged). In this solution the various parameters are found to obey

$$a^2 \sinh^2\gamma + \kappa_5^2\nu^2/3 = 1, \quad -\kappa_5^2V_0/6 = 1, \quad \Lambda_b^2\kappa_5^2/6 + V_0 = 6a^2/(\kappa_5^2 - \kappa_5^4\nu^2/3), \quad (12)$$

to yield a structure very similar to that exhibited in Eq. (8). Despite the fact that our potential is unbounded from below, the quantity $e^{2f}[\phi'^2/2 + V(\phi)]$ remains bounded because of compensating zeroes in the metric. Thus while a potential such as that of Eq. (11) could not be considered in flat space physics, in the event of curvature it is possible for the energy

¹⁵The general (Minkowski and Euclidean) signed 2-dimensional metrics $ds^2 = dx^2[1 + \cosh\theta(x, y)] \pm dy^2[1 - \cosh\theta(x, y)]$ will be spaces of constant 2-curvature K provided θ obeys the conditions $\partial^2\theta/\partial_x^2 \mp \partial^2\theta/\partial_y^2 = -2K\sinh\theta$, and will support the y -independent mode $\tanh(\theta/2) = \sin[(-2K)^{1/2}x]$ if K is negative, a mode in which θ becomes infinite at finite x .

density in the gravitational field to compensate and make the theory well behaved. With such a diverging of ϕ at the compactification points we thus uncover a mechanism which actually serves to disconnect the $|w| < \gamma$ and $|w| > \gamma$ regions and thereby force compactification upon us in the first place. Moreover, we can even consider the points $\phi = \pm\infty$ as being effective "minima" of the theory since they are the points at which $V(\phi)$ takes its lowest values within the compactification region; with the point $\phi = 0$ at which the brane is located being a local maximum, one about which gravity is then localized.

While we have not been able to find a closed form generalization of this solution beyond the brane limit, or found any solution at all that that would lead to Eq. (10) in the same way that Eq. (6) led to the R_4 RS limit, nonetheless the existence of a connection between scalar field theory and a dS_4 RS brane in the limit strongly suggests the existence also of such a connection even before the brane limit is taken. We are indebted to Drs. M. Gremm and K. Berhndt for notifying us of their work following the release of an earlier version of this manuscript. P. D. Mannheim would like to thank Drs. R. L. Jaffe and A. H. Guth for the kind hospitality of the Center for Theoretical Physics at the Massachusetts Institute of Technology where part of this work was performed. The work of P. D. Mannheim has been supported in part by funds provided by the U.S. Department of Energy (D.O.E.) under cooperative research agreement #DF-FC02-94ER40818 and in part by grant #DE-FG02-92ER40716.00.

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